

A Noise-Assisted Reprogrammable Nanomechanical Logic Gate

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ABSTRACT We present a nanomechanical device, operating as a reprogrammable logic gate, and performing fundamental logic functions such as AND/OR and NAND/NOR. The logic function can be programmed (e.g., from AND to OR) dynamically, by adjusting the resonator's operating parameters. The device can access one of two stable steady states, according to a specific logic function; this operation is mediated by the noise floor which can be directly adjusted, or dynamically "tuned" via an adjustment of the underlying nonlinearity of the resonator, i.e., it is *not* necessary to have direct control over the noise floor. The demonstration of this reprogrammable nanomechanical logic gate affords a path to the practical realization of a new generation of mechanical computers.

KEYWORDS Nanomechanical logic, nanomechanical computing, logical stochastic resonance, stochastic resonance, nanomechanical resonator

A practical realization of a nanomechanical logic device, capable of performing fundamental logic operations, is yet to be demonstrated despite a long-standing effort toward scalable mechanical computation.^{1–4} This effort can be traced back to 1822 (at least), when Charles Babbage presented a mechanical computing device that he called the "Difference Engine", to the Royal Astronomical Society.¹ Before this event, though, the search for mechanical computing devices had already been inherent in attempts to build machines capable of computation. This search has, today, taken on added urgency as we seek to exploit emerging techniques for the manipulation of matter at nanometer length scales. With Boole's ideas on logic operations with two states an added dimension to computing, logic elements or gates have come to dominate modern computation. However mechanical logic, especially at the very small length scales and in the presence of a noise floor, has proven difficult to realize despite some recent experimental efforts.^{5–7}

The control and manipulation of mechanical response at nanometer scales can be realized by exploiting a (seemingly) counterintuitive physical phenomenon, stochastic resonance (SR):⁸ in a noisy nonlinear mechanical system, the controlled addition of noise can enhance the system response to an external stimulus. Signal amplification in such a setup has been experimentally realized in nonlinear nanomechanical resonators configured as two-state devices.^{9–11} Recently, it has been demonstrated¹² that when two square waves are

applied as input stimuli to a two-state system, the response can result in a specific logical output with a probability (for obtaining this output) controlled by the noise intensity. Furthermore, changing the threshold (either via adjusting the nonlinearity strength or applying a controlled asymmetrizing dc signal) can change or "morph" the system output into a different logic operation.¹²

Our experimental logic device consists of a nanomechanical resonator, operating in the nonlinear regime, wherein two different vibrational states coexist; for an underdamped system underpinned by an *a priori* monostable (but nonparabolic) potential energy function, these vibrational steady states are induced by biasing the system with a high-frequency (typically several megahertz) controllable sinusoidal drive signal. Then, the effect of the drive is to tilt the potential energy function, thereby creating left and right steady states, whose (average) lifetimes depend on the interplay between the drive amplitude, damping coefficient, and the root mean square intensity of the system noise floor. These two states form the basis for performing binary logic by defining the logic value of the output.^{9,11,13,14} The logic inputs are physically implemented by two square waves of fixed level that are electrically added and applied to the resonator. These inputs produce a modulation of the resonator's frequency which, in the presence of stochastic noise, can induce switching between the vibrational states, thus changing the logic value of the output. By variation of the applied noise power, an optimal window of noise is found wherein the output is a predefined logical function of the inputs. Furthermore, the logic function can be dynamically changed from one operation to another by adjusting the resonator's operating point; effectively, we change the drive

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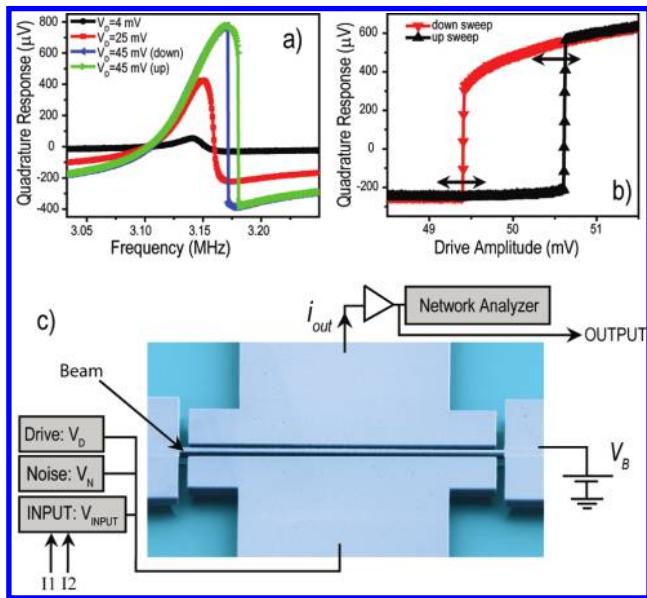


FIGURE 1. (a) Response in quadrature of the resonator as a function of the driving frequency for different values of the drive amplitude. For small drive amplitudes the resonator exhibits the usual Lorentzian line shape. As the drive amplitude is increased, the resonance shifts toward higher frequency until the bistable regime is reached. (b) Response in quadrature of the resonator as a function of the drive amplitude for a fixed frequency (3.158 MHz) in the bistable regime. The response shows the usual hysteretic behavior when the drive amplitude is swept in different directions (increasing amplitude up triangles, decreasing amplitude down triangles). The horizontal arrows represent the effect of the input signal. In the absence of noise this modulation is not able to produce switching between the two states, but as noise is added switching between the two states inside the hysteretic regime becomes possible. The arrow on the top right represents the NOR/OR situation while the arrow on the bottom left represents the NAND/AND situation. (c) Micrograph of the resonator and experimental setup. A network analyzer is used to drive the resonator at the desired frequency and amplitude, while a signal generator is used to produce “white” noise in a 100 kHz band encompassing the resonator’s resonance. A second signal generator provides the input signal ($I_1 + I_2$). Due to the nature of the actuation scheme, the input signal is mixed by the resonator producing a frequency modulation. The output current is amplified by a transimpedance amplifier and measured with the vector network analyzer set to continuous wave (CW) time mode; it measures the time dependence of the resonator amplitude and phase at the drive frequency. The beam deflection is in the 1–5 nm range.

amplitude while keeping the drive frequency constant, so that a reoptimization of the noise floor is not necessary.

The resonator was fabricated from single-crystal silicon using standard e-beam lithography and surface nanomachining. It consists of a doubly clamped beam with two adjacent electrodes (Figure 1) used to actuate and detect the in-plane motion of the beam using standard room temperature electrostatic techniques.¹⁵ The beam is 20 μm long, 300 nm wide, and 500 nm thick. The gap (g) between the beam and the electrodes is 250 nm. At room temperature, the nanomechanical beam demonstrates the expected normal mode with a resonance frequency $f_0 = 3.145$ MHz and a quality factor $Q = 70$ (at a pressure ~ 10 mTorr).

To drive the resonator, a high-frequency voltage of amplitude V_D (ω) is applied to one of the electrodes. This produces the in-plane motion of the beam and, hence, the

modulation of the capacitance between the beam and the detection electrode (C). In the presence of a dc bias voltage applied to the beam ($V_B = 14$ V), the time-dependent modulation of the capacitance results in a current $i_{out} = \dot{x}V_B(dC/dx)$, where x is the effective displacement of the beam. The capacitances between the beam and the electrodes can be modeled through parallel plate capacitors. Since the displacement is much smaller than the gap between the beam and electrodes, and the drive amplitude and the amplitude of the input signal are much smaller than the bias voltage, one obtains for the dynamics of the system

$$\ddot{x} + \gamma\dot{x} + \omega_0^2x + k_3x^3 = f_D(t) + f_{\text{INPUT}}(t)\frac{x}{g} + f_N(t) \quad (1)$$

Here, γ is the dissipation coefficient, $\omega_0/2\pi$ is the resonance frequency, k_3 is a nonlinear spring constant, f_D is the driving force (corresponding to the drive voltage V_D), f_N is the force due to the applied white noise, and f_{INPUT} is the force due to the input. It is noteworthy that the input force term is multiplicative, which implies that this term is only observable when the frequency of the input is within the bandwidth of the resonator.¹⁵ Equation 1, in the absence of noise and input, predicts the standard behavior of the resonator as a function of the drive amplitude,^{13,16} going from the linear to the bistable regime, as shown in Figure 1. At any given frequency in the hysteretic bistable regime, the resonator can exist in two distinct amplitude states, separated by a potential barrier; these two states can be used as a binary element. Logic operations on this binary element are accomplished by choosing the appropriate inputs to the electrostatic gates to the resonator.

The logic inputs, represented by two asynchronous square waves (I_1, I_2) of amplitude 12.5 mV_{pp} are electrically added ($I = I_1 + I_2$) and applied to the actuation electrode. The logic values 1 and 0 are represented by the high and low level of each of the inputs. When these signals are added, they give rise to three different voltage levels depending on the logic value: (1,1) has a voltage of 25 mV, (1,0) and (0,1) a voltage of 0 mV, and finally (0,0) –25 mV. The two vibrational states of the resonator are defined as the two states of the single output of the logic element.

Switching between the two output states can be accomplished by a modulation signal applied to the drive. Beyond a threshold value, switching between the states can coherently follow the modulation. In the subthreshold regime, coherent switching (in response to the modulation signal) between the states is mediated by the noise.

The points of operation of the resonator are the edges of the bistable region, as shown in Figure 1. When the input is applied the operation point moves, as marked by the horizontal arrows in Figure 1b. At the operating point,

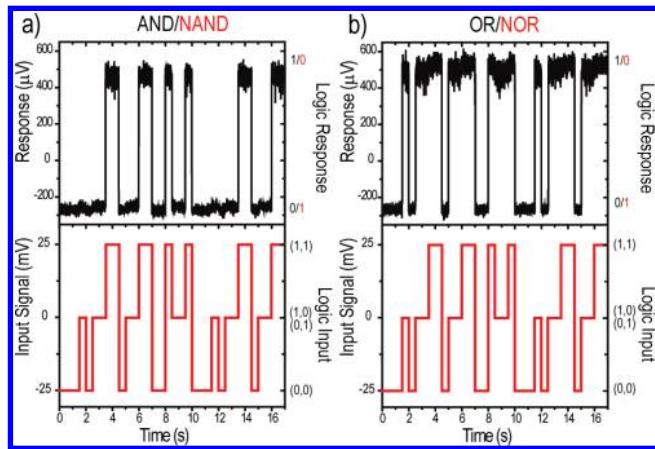


FIGURE 2. (a) (top) Response in quadrature of the resonator to the low-frequency input (bottom) as a function of time in the presence of white noise showing AND/NAND logic. The drive frequency is 3.158 MHz and the drive amplitude is 49.3 mV with a noise power of -89 dB m. The input signal (I) is an aperiodic three-state square wave with amplitude of 50 mV_{pp}, equivalent to the electric sum of two aperiodic square waves (I_1 and I_2) with amplitudes of 25 mV_{pp}. Top left side indicates the logic response of the resonator. Depending on the assignment of the logical value to the vibrational states of the resonator an AND (black) gate or NAND (red) gate is obtained. Bottom left side indicates the logic values of the inputs written as (I_1, I_2). Note that the logic states (1,0) and (0,1) correspond to the same electrical level. (b) Response in quadrature of the resonator to the low-frequency input (bottom) as a function of time in the presence of white noise showing OR/NOR logic. In this case the drive frequency and noise power are the same as in (a) but the drive amplitude is 50.5 mV.

controlled switching can be induced between the two states in the presence of noise.

The noise-induced coherent switches are only produced when the input has the “correct” value as shown in Figure 2; in other words, the logic function of the device (e.g., AND/ OR, or NAND/NOR) is defined by choosing the “correct” input. For the AND/NAND gate (arrow bottom left of Figure 1b), the input can only produce a switch from the low level state to the high level state (logic output 1) when it has a value of 25 mV, corresponding to a logic input (1,1). For the OR/NOR gate (arrow top right of Figure 1b), the switch from the high to the low level state is only accomplished when the input value is -25 mV, logic (0,0).

In both cases, the reliable logic gate is realized for an optimal level of noise. For low noise power, switches are synchronized with the input but they are sporadic, as shown in Figure 3 (top). As the noise power is increased, the optimal noise window is reached (Figure 3, middle) where the output is the desired logic function of the inputs with probability equal to 1. With a further increase in the noise power, random switches begin to occur, destroying any logic relation between input and output (Figure 3, bottom). These observations can be quantified by calculating the probability of obtaining the desired logic function. The results are shown in Figure 4 for both the AND/NAND and OR/NOR cases. The probability is equal to 1 for both types of gates in the same noise window. This is a necessary condition since in a

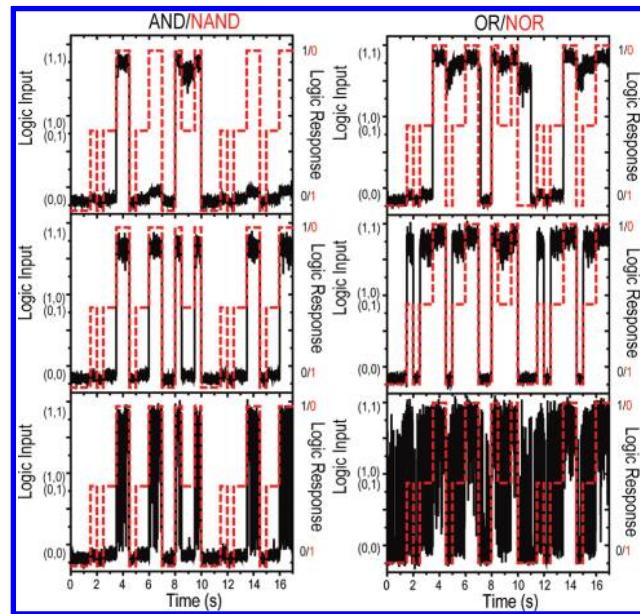


FIGURE 3. Progression of the response of the resonator in the presence of white noise (increasing from the top). Left (right) side shows the resonator performing AND/NAND (OR/NOR) logic. For low noise power (-93 dB m) the input is not able to produce reliable transitions between the two states. As the noise is increased, an optimal noise power is reached (-89 dB m) in which the resonator switches synchronously with the input, obtaining in this way a reliable logic gate. Further increase of the noise power (-82 dB m left, -79 dB m right) leads to the occurrence of random switches, destroying the reliability of the logic gates. The drive conditions for the resonator are the same as in Figure 2.

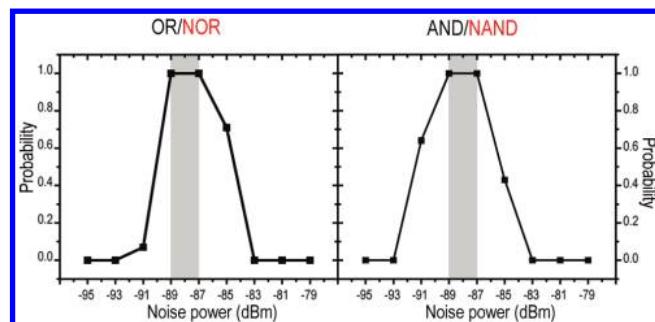


FIGURE 4. Left, probability of obtaining an OR/NOR logic gate as a function of noise power. Right, probability for an AND/NAND logic gate as a function of noise power. The probability is calculated using an input signal with all the possible values, repeated 30 times (a total of 90 logic operations are performed). If the output matches the expected logic output for all possible combinations is considered a success. The probability is then calculated as the number of successes divided by the total number of attempts¹² (in this case 30). The drive conditions for the resonator and noise level applied are the same as in the central panel of Figure 3.

realistic application noise power may not be controlled, which makes drive amplitude and, e.g., in this experimental setup, the nonlinearity strength (governed by the natural frequency ω_0 and/or the coefficient k_3) the only tunable parameters to reprogram logic response. We emphasize, however, that adjusting the nonlinearity is tantamount to dynamically “tuning” the internal noise floor.¹⁷ The addition

of a small (and controllable) dc signal to the input can also tune (in this case, asymmetrize, at time $t = 0$) the transfer characteristic and, hence, the internal noise floor. In any case, it is *not* necessary (and, often, unrealistic in real applications) to directly control the system noise.

An important measure of the device performance is the energy cost of a single logic operation. This can be estimated as the change in energy stored in the system due to the applied voltage necessary to switch the state of the resonator, $E_{\text{diss}} = CV_B V_I \sim 10^{-17}$ J, where V_I is the input voltage (25 mV, in this case). It is well-known that the minimum energy dissipated by an irreversible logic operation is limited by the heat released due to the loss of one bit of information,¹⁸ $k_B T \ln 2$. For the present realization, the relevant noise source is the externally applied white noise¹⁹ with $k_B T_{\text{eff}} \sim 10^{-18}$ J (corresponding to a total noise power of -88 dB m), where T_{eff} is the effective temperature of the applied noise. This results in a dissipated energy very close to the Landauer limit $E_{\text{diss}} \sim 10k_B T_{\text{eff}}$. It is important to note that $T_{\text{eff}} \approx 10^3 T$ at room temperature. The power needed for the operation of this prototype reprogrammable logic gate is ~ 0.1 nW in the low level state and ~ 0.3 nW in the high level state. We estimate it to be $V_D i_{\text{out}}$, where V_D is the drive amplitude and i_{out} is the current produced by the resonator before amplification. Minimum required power can be reduced by orders of magnitude in the next generation of devices by optimizing the device geometry and actuation mechanisms.

The speed of operation is governed by the noise induced switching rate, Γ . In this case the measured rise and fall time are 0.2 ms (the rise and fall time for the input is of the order of nanoseconds). This is in good agreement with previously measured transition rates in similar devices.¹⁰ The transition rate is given²⁰ by $\Gamma \approx (\omega_0/2\pi Q)e^{(-E_A/k_B T_{\text{eff}})}$, with $E_A \propto (\omega - \omega_c)^2$, where ω is the drive frequency and ω_c is the critical frequency (frequency at which the bifurcation takes place for a given drive amplitude). Hence, there are many ways of improving the operation speed—i.e., by increasing the resonance frequency of the resonator, increasing the noise power, or simply changing the drive frequency.

The realization of nanomechanical logic gates with power consumption and size competitive with the current CMOS logic gates is exciting. It not only enables a path toward an alternative architecture beyond the limit to which current microprocessors can be scaled but also provides a fundamental building block for alternate computing schemes apart from the straightforward swap-in with the conventional logic gates. For instance, (stochastic) noise-assisted nanomechanical logic elements can be used for direct and controlled computing to harness nonlinearity and exploit inherent parallelism. Such approaches have already been shown to result in highly desirable architectures using flexible parallel implementations of chaotic logic gates.²¹ It merits comment

that, in this work, we appear to have achieved a fusion of both Babbage's and Boole's visions with today's nanotechnology.

Nanomechanical logic elements described here are three-dimensional structures, which enable local inputs, outputs, and controls. Furthermore, due to the geometrical flexibility, it is possible to include additional constraints such as symmetry in inputs and outputs toward the realization of elements for reversible computation such as Feynman's billiard ball logic or the Fredkin–Toffoli gate.²² An even more fascinating possibility arises when the nanomechanical resonator is operated in a regime of high frequency and low temperature so that its energy levels are quantized, with the two-level structure corresponding, now, to two quantized energy states. While the system discussed in this work was not operated in this limit, there has, already, been speculation^{9,23} regarding the potential applicability of the “stochastic resonance” effect to quantum measurement and control scenarios.

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